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Controlling traffic jams by a feedback signal

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Abstract. The aim of this paper is to control the vehicle system to move orderly by a feedback signal in the optimal velocity (OV) model under open boundary. Following the description of the OV model and its stability analysis, some problems about the definition of traffic jams in [12] are presented and illustrated by an example. Based on our analysis, a feedback signal acting on the traffic system has been introduced into the OV model. The signal will act in the system only if the state is inhomogeneous, and it will vanish in the homogeneous traffic flow. Theoretically, it is proved that the disorder state in traffic system could be suppressed by using the control method. Two kinds of noises are tested in our computer simulations. The simulation results demonstrate that the traffic system can move into a homogeneous phase by the control.

PACS. 07.05.Dz Control systems – 05.70.Fh Phase transitions: general studies – 89.90.+n Other topics in areas of applied and interdisciplinary physics

1 Introduction

In recent years, traffic problems have been widely investigated [1–10]. In order to understand the complex traffic behavior, several traffic flow models have been developed, including macroscopic models where traffic is viewed as a compressible fluid formed by the vehicles, and microscopic models where individual vehicle is represented by a particle and the vehicle traffic is treated as a system of interacting particles driven far from equilibrium. Most of the works were focused on the traffic jam transitions and the mechanism of the traffic jam phenomena [6–10]. It is well known that traffic jams bring not only traffic safety problems but also environment pollution problems in traffic flow. Therefore, the solution of traffic jams is very important, which incites us to use new method to avoid the traffic jams.

In many works, the jamming transitions and characteristics of traffic jams have been investigated in the optimal velocity (OV) model [11–13]. The OV model proposed by Bando et al. [7], is a favorable one of the microscopic traffic models and has been studied in great detail by using the numerical and analytical methods. In this model, each vehicle is described by a simple differential equation using OV function, which is dependent on the headway distance, and each driver controls the velocity based on the OV function. Konishi et al. [11,12] investigated the traffic jams phenomena under periodic boundary condition and derived a simple stability condition of the OV model. Komatsu and Sasa [13] studied the traffic jams on the OV model in detail. However, there are seldom studies on controlling vehicle traffic system from jams to homogeneous states. In the present study, a feedback signal is utilized to control the vehicle traffic from jams to order states. The paper is organized as follows: the OV model and its stability analysis are reviewed in Section 2. And some problems about the definition of traffic jams proposed by Konishi et al. [12] are presented and illustrated by an example. We introduce the controlling method and give its theoretical analysis in Section 3. Our simulations are shown in Section 4. The final section is our conclusions and future works.

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2 Optimal velocity model

2.1 Description of model

Our investigations are based on the optimal velocity (OV) model under open boundary condition. The leading vehicle is described as follows [7]

$$x_1(t) = v_0 t + x_1(0) \tag{1}$$

where $x_1(t) > 0$ is the position of the leading vehicle at time t, $v_0 > 0$ is its velocity which is a constant at any time, and $x_1(0) > 0$ is its initial position.

The following vehicle is described by the following equation of the motion of vehicle i [7]

$$\frac{d^2 x_i(t)}{dt^2} = a\left[A(\Delta x_i(t)) - \frac{dx_i(t)}{dt}\right]$$
$$\Delta x_i(t) = x_{i-1}(t) - x_i(t), \quad i = 2, 3, \ldots, N \tag{2}$$

where $x_i(t) > 0$ is the position of the ith vehicle at time t, $\Delta x_i(t)$ is the headway distance between the ith vehicle and its front vehicle $(i-1)$th vehicle at time t, a is the sensitivity of the driver, and N is the total number of vehicles.

A driver adjusts the car velocity to approach the optimal velocity, which is determined by the observed headway distance. The sensitivity a allows for the time lag $1/a$ that makes the car velocity reach the optimal velocity when the traffic is varying. Generally, it is necessary that the optimal velocity function has the following properties: it is a monotonically increasing function. The optimal velocity has been given by [7]

$$A\left(\Delta x_i(t)\right) = \tanh\left(\Delta x_i(t) - x_c\right) + \tanh(x_c)$$

where x_c is the safety distance, $\tanh(\cdot)$ is the hyperbolic tangent function.

2.2 Stability analysis

Konishi et al. [12] rewrote the dynamical equation (2) of the following vehicles as

$$\frac{dv_i(t)}{dt} = a\left[A(\Delta x_i(t)) - v_i(t)\right]$$
$$\frac{d\Delta x_i(t)}{dt} = v_{i-1}(t) - v_i(t), \quad i = 2, 3, \ldots, N. \tag{3}$$

Assume the lead vehicle runs constantly with speed v_0, and then the following vehicles have the following steady states [12]:

$$\left[v_i^*(t), \Delta x_i^*(t)\right]^T = \left[v_0, A^{-1}(v_0)\right]^T. \tag{4}$$

After linearizing the vehicle system (3) around steady state (4), the relation between the $(i-1)$th vehicle velocity disturbance and the ith vehicle velocity disturbance is obtained [12]

$$V_i(s) = G(s)V_{i-1}(s) \tag{5}$$

where $V_i(s) = \mathcal{L}(\bar{v}_i(t)), V_{i-1}(s) = \mathcal{L}(\bar{v}_{i-1}(t))$, $\mathcal{L}(\cdot)$ denotes the Laplace transform (the Laplace transform is an integral transform perhaps second only to the Fourier transform in its utility in solving physical problems) [14,16]

$$\bar{v}_i(t) = v_i(t) - v_0, \quad \bar{v}_{i-1}(t) = v_{i-1}(t) - v_0.$$

The transfer function $G(s)$ is

$$G(s) = \frac{a\Lambda}{d(s)}$$
$$\Lambda = \left.\frac{dA(\Delta x_i(t))}{d(\Delta x_i(t))}\right|_{\Delta x_i(t) = A^{-1}(v_0)}$$
$$= \frac{1}{\left[\cosh(A^{-1}(v_0) - x_c)\right]^2} \tag{6}$$

where $\cosh(\cdot)$ is the hyperbolic cosine function.

The characteristic polynomial $d(s)$ is

$$d(s) = s^2 + as + a\Lambda.$$

According to control theories, Konishi et al. directly derived a simple definition about traffic jams occurring for the OV traffic model [12]. The definition is given as follows [12].

Definition 1. Assume that characteristic polynomial $d(s)$ is stable. If $H_\infty - norm$ of $G(s)$ is greater than 1, that is

$$\|G(s)\|_\infty = \sup_{\omega \in [0, +\infty)} |G(j\omega)| > 1$$

then traffic jam occurs in the OV traffic model.

Here a real polynomial $d(s)$ is said to be stable if all its roots lie in the left half-plane. The term "stable" is used to describe such a polynomial because, in the theory of linear servomechanisms, a system exhibits unforced time-dependent motion of the form e^{st}, where s is the root of a certain real polynomial $d(s)$. A system is therefore mechanically stable if $d(s)$ is a stable polynomial [15,16].

From Definition 1, Konishi et al. pointed out that the traffic jam never occurs when the following two conditions are satisfied: (1) $\|G(s)\|_\infty \le 1$; (2) $d(s)$ is a stable polynomial. From control theory view, the definition for traffic jam is no problem. However, traffic system, which has its special characteristics, cannot simply be treated as a control system. We will discuss some problems about Definition 1 in detail in the following section.

2.3 Some problems about Definition 1

In general, there are two kinds of disturbance for real traffic system (a disturbance is a signal which causes some variations in the normal condition of the original system), including helpful disturbances, which are desirable for traffic systems, and harmful disturbances, which may lead to traffic jams in most cases. Therefore, Definition 1 is unsuitable for helpful disturbances, which is still viewed as a normal signal. When $\|G(s)\|_\infty > 1$, the signal will amplify upstream. And the larger the helpful signal becomes, the better the traffic system runs. There is no traffic jam in this case, although $\|G(s)\|_\infty$ is greater than 1. Such example will be considered on computers.

The simulations are carried out in the OV model under open boundary. The vehicles system in a road with the length of 200 m is studied. The parameters are set as $a = 1.0$, $x_c = 2.0, v_0 = 0.964$, which is same as paper [12]. In this case, the characteristic polynomial $d(s)$ is stable and $\|G(s)\|_\infty > 1$. Fourth-order Runge-Kutta method is used for numerical integration with time step $\Delta t = 1/128$ in the OV model. The system has no car at initial state. From the left side, the upper stream, a car is injected with a probability $r = 0.6$ every second if the distance between the left side and the tail of the sequence of cars is larger than x_c. The initial speed of the injected car is zero.

We show the spatio-temporal pattern of the traffic flow in the OV model with a disturbance. The external step disturbance is added as the following:

$$v_1(t) = \begin{cases} v_0, & \text{if} \quad 0 \le t \le 500 \\ \frac{3}{2}v_0, & \text{if} \quad 500 < t \le 1000. \end{cases}$$

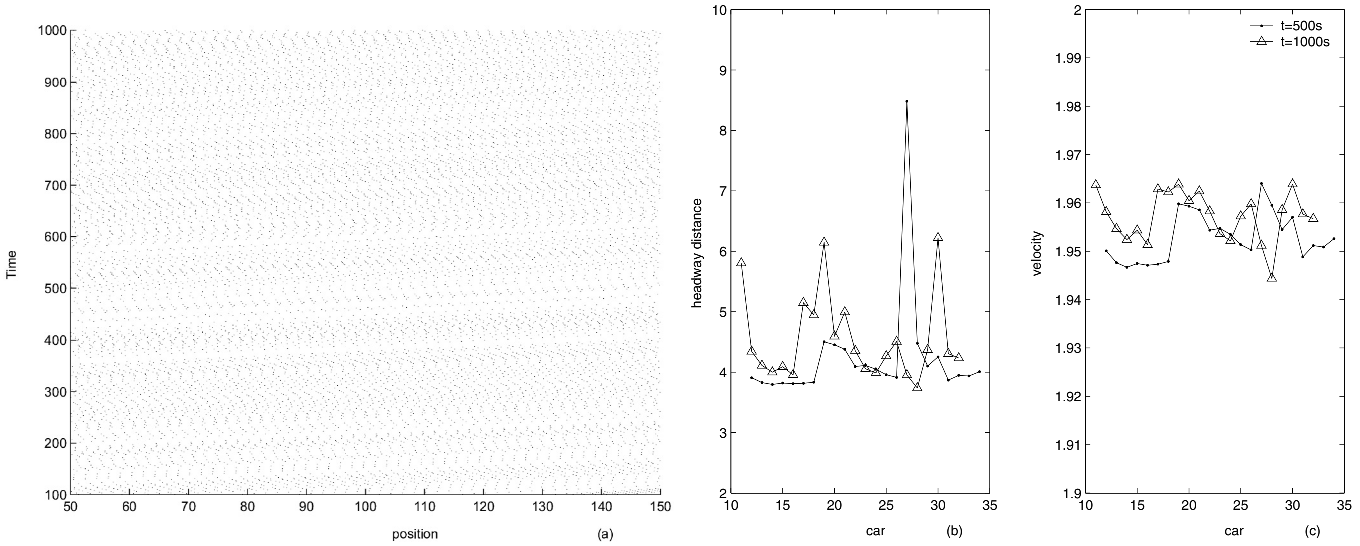

Fig. 1. (a) Space-time diagram in the OV model with a helpful disturbance ($t > 500$); (b) headway distance at $t = 500$ s and $t = 1000$ s corresponding to Figure 1a; (c) velocity at $t = 500$ s and $t = 1000$ s corresponding to Figure 1a.

Obviously, external step disturbance is helpful to the traffic system. Figure 1a shows the local space-time diagram of the OV model. To avoid transient state, two parts on the road, the upstream 50 m and the downstream 50 m, are discarded. From Figure 1a, it is found that there is no obvious high-density region after the disturbance is added. In order to get information on the spatial organization of the vehicles, the variations of corresponding distance-headway and velocity at $t = 500$ and $t = 1000$ are placed in Figures 1b and 1c, respectively. Without the disturbance, the vehicles freely run on the road. Compared with no disturbance traffic system, there are larger headway distances between two successive vehicles to make the vehicle system move freely with larger speed in the OV model with the step disturbance. Furthermore, the simulation data show that the helpful disturbance brings the increase of the flux in the traffic system. The results demonstrate that traffic flow is at free state after the step disturbance is introduced. That means in this case there is no traffic jams in the disturbed OV model when $\|G(s)\|_\infty > 1$ and the characteristic polynomial $d(s)$ is stable. However, two noisy traffic systems under the two conditions of Definition 1 in [12] appear traffic jams. Therefore, it is uncertain whether traffic jams occur in the OV models when the two conditions of Definition 1 are satisfied. That is, the two conditions given by Definition 1 are not sufficient and necessary for traffic jams.

In addition, it is can be sure that the traffic jams never occur when $\|G(s)\|_\infty \le 1$ and $d(s)$ is a stable polynomial. Whether the disturbance is helpful or harmful to the vehicles system, it will decay upstream and vanish at the end.

In conclusions, it is certain that there is no traffic jams in the OV model if $\|G(s)\|_\infty \le 1$ and $d(s)$ is a stable polynomial; it is uncertain that whether traffic jams occurs or not if $\|G(s)\|_\infty > 1$ and $d(s)$ is a stable polynomial. It is necessary to redefine the concept about traffic jams. In this paper, based on the analysis, a feedback control signal is designated to suppress the traffic jam in the OV model.

3 Feedback control

A feedback control signal term $u_i(t)$ is designated to add to vehicle dynamics (2):

$$\begin{aligned} \frac{dv_i(t)}{dt} &= a\left[A(\Delta x_i(t)) - v_i(t)\right] + u_i(t) \\ \Delta x_i(t) &= x_{i-1}(t) - x_i(t), \quad i = 2, 3, \ldots, N \end{aligned} \tag{7}$$

where the control signal $u_i(t)$ is:

$$u_i(t) = k(v_{i-1}(t) - v_i(t)) \tag{8}$$

k is the feedback gain, which is adjustable. The control signal $u_i(t)$ is proportional to the difference between the velocity $v_{i-1}(t)$ in front of vehicle i and its velocity $v_i(t)$ in time t. It is noted that the control signal acts in the system only if the state is unstable, and the signal vanishes in the stable system.

The control system (7) can be linearized at the steady state (4), that is

$$\begin{aligned} \begin{bmatrix} \dfrac{d\bar{v}_i(t)}{dt} \\ \dfrac{d[\Delta\bar{x}_i(t)]}{dt} \end{bmatrix} &= \begin{bmatrix} -a & a\Lambda \\ -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \bar{v}_i(t) \\ \Delta\bar{x}_i(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \bar{v}_{i-1}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot u_i(t), \\ \bar{v}_i(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \bar{v}_i(t) \\ \Delta\bar{x}_i(t) \end{bmatrix}, \\ u_i(t) &= k[v_{i-1}(t) - v_i(t)] = k[\bar{v}_{i-1}(t) - \bar{v}_i(t)] \end{aligned} \tag{9}$$

where $\Delta \bar{x}_i(t) = \Delta x_i(t) - \Lambda^{-1}(v_0)$.

From frequency domain viewpoint, this linearized system can be written as

$$V_i(s) = G_{11}(s)V_{i-1}(s) + G_{12}(s)U_i(s)$$
$$U_i(s) = k[V_{i-1}(s) - V_i(s)] \quad (10)$$

where

$$G_{11}(s) = \frac{a\Lambda}{d(s)}, G_{12}(s) = \frac{s}{d(s)}. \quad (11)$$

The relation between $V_{i-1}(s)$ and $V_i(s)$ is described as

$$V_i(s) = \bar{G}(s) \cdot V_{i-1}(s). \quad (12)$$

Substitute equation (10) with equation (11), $\bar{G}(s)$ is given by

$$\bar{G}(s) = [G_{11}(s) + k \cdot G_{12}(s)] \cdot [1 + kG_{12}(s)]^{-1} = \frac{a\Lambda + ks}{\bar{d}(s)} \quad (13)$$

where

$$\bar{d}(s) = d(s) + ks = s^2 + (a + k)s + a\Lambda.$$

It is noted that equation (12) corresponds to equation (5). According to our analysis, the traffic jams will never occur in the controlled system if the characteristic polynomial $\bar{d}(s)$ of $\bar{G}(s)$ is stable and the $\|\bar{G}(s)\|_\infty \leq 1$.

No traffic jams can be realized by adjusting the parameter k in the controlled system. Firstly, according to Hurwitz stability criterion, $\bar{d}(s)$ is stable if

$$\begin{cases} a + k > 0 \\ a\Lambda > 0. \end{cases} \quad (14)$$

Equation (14) equals to $k > -a$, because of the OV function characteristics of monotonously increase (namely $\Lambda \geq 0$) and $a > 0$. Therefore, the first condition for the feedback gain k is given as

$$k > -a. \quad (15)$$

Secondly, consider $\|\bar{G}(s)\|_\infty \leq 1$, that is

$$\|\bar{G}(s)\|_\infty = \sup_{\omega \in [0,+\infty)} |\bar{G}(j\omega)| = \sup_{\omega \in [0,+\infty)} \sqrt{\frac{a^2\Lambda^2 + k^2\omega^2}{(a\Lambda - \omega^2)^2 + (a + k)^2\omega^2}} \leq 1. \quad (16)$$

Equation (16) is tenable if the following equation is fulfilled

$$\frac{a^2\Lambda^2 + k^2\omega^2}{(a\Lambda - \omega^2)^2 + (a + k)^2\omega^2} \leq 1. \quad (17)$$

That is,

$$a^2\Lambda^2 + k^2\omega^2 \leq (a\Lambda - \omega^2)^2 + (a + k)^2\omega^2$$
$$(a^2 + 2ak - 2a\Lambda)\omega^2 + \omega^4 \geq 0. \quad (18)$$

The sufficient condition for equation (18) is given as

$$a^2 + 2ak - 2a\Lambda \geq 0$$
$$k \geq \Lambda - \frac{a}{2}. \quad (19)$$

Therefore, the second condition for the feedback gain k is given as equation (19).

From the analysis above, we can infer that

Theorem 1. If the condition of (15) and (19) are satisfied, when a feedback signal (8) is added to the vehicle system (2), traffic jams will never occur in the controlled system (7).

For the given a and OV function, it is easy to determine the value of the feedback gain k. The traffic jams are suppressed by the feedback signal (8), and the vehicles move forward orderly with the control signal.

Since the space-time diagram can macroscopically manifest the state of traffic flow, space-time diagram is used to show spatio-temporal pattern of traffic flow. The variation of the distance headway and velocity are considered to observe the variation of traffic state, also.

4 Simulation results

Our simulations are carried out in the same condition as paper [12]. There are 100 vehicles running on a single road under an open boundary. The road length is 200 m. The parameters are set as $a = 1.0$, $x_c = 2.0$, $v_0 = 0.964$, which are the same as the previous paper [12]. Fourth-order Runge-Kutta method is used for numerical integration with time step $\Delta t = 1/128$ in the OV model. The initial conditions are chosen as $\Delta x_i(0) = x_c$, $i = 2, \ldots, 100$, $v_i(0) = \Lambda^{-1}(x_c) = v_0$.

Firstly, we show the spatio-temporal pattern of the traffic flow induced by the noise in the OV model without control. Figure 2a shows the space-time diagram of the OV model with noise. The dimensionless noise, added to the first equation of (2) for all vehicles, is assigned by generating the random number with maximum amplitude 10^{-3}. The generating noise is harmful to the traffic system. The horizontal axis is defined as follows

$$\bar{x}_i(t) = L + x_i(t) - x_1(t), \quad i = 1, 2, \ldots, N.$$

It represents a distance between the leading vehicle and the following vehicle and the leading vehicle is fixed at 200 m. The vertical axis indicates the time development. Simulations are started from a homogeneous initial condition, and the first 100 data are discarded to avoid transient. From Figure 2a, it can be found that the diagram exhibits macroscopic phase segregation into disorder region

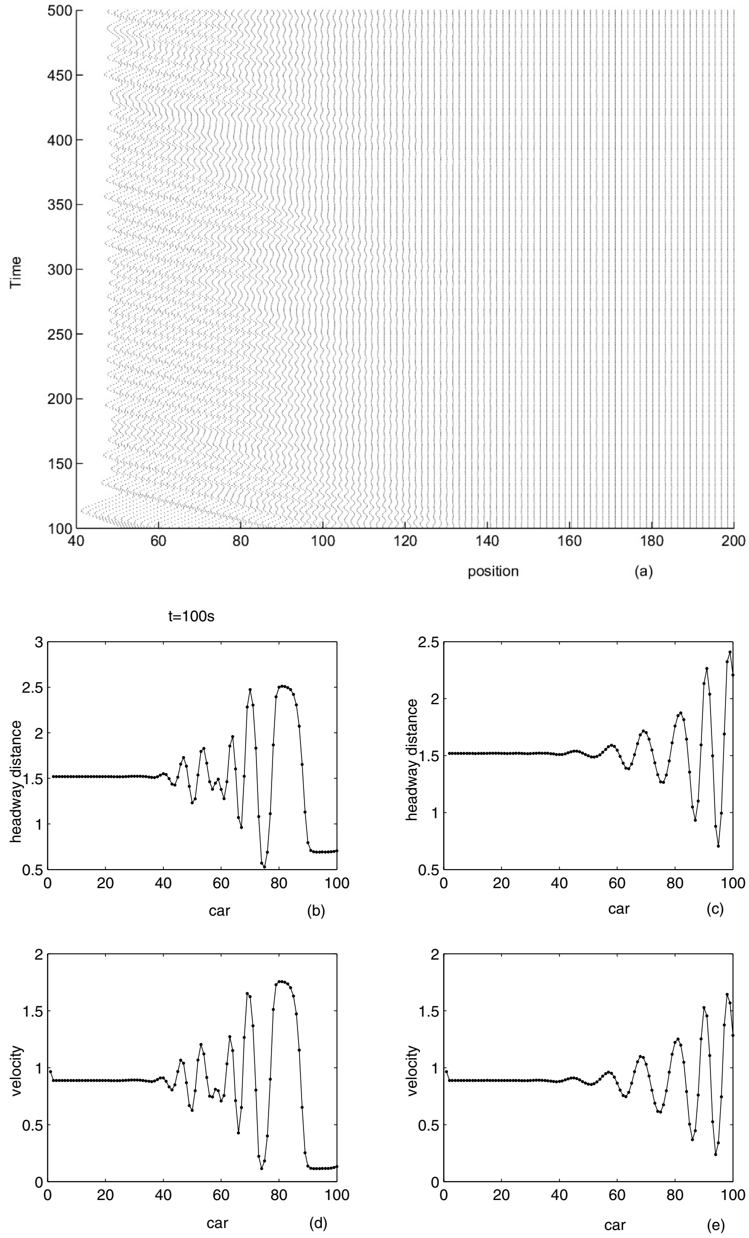

Fig. 2. (a) Spatio-temporal pattern of the traffic flow in the noisy OV model without control ($t = 100 \sim 500$); (b-c) headway distance at $t = 100$ s and $t = 500$ s corresponding to Figure 2a; (d-e) velocity at $t = 100$ s and $t = 500$ s corresponding to Figure 2a.

upstream and order region downstream. In order to get information on the spatial organization of the vehicles, the variations of corresponding distance-headway at $t = 100$ and $t = 500$ are placed in Figures 2b and 2c, respectively. In Figures 2b-c, it is obvious that the distance-headway is constant downstream, and gradually oscillates around the constant headway distance upstream. The oscillation amplitude increases with the increase of car index. Figures 2c-d shows the velocity behavior of all vehicles at $t = 100$ and $t = 500$. From Figures 2c-d, it is can be seen that the upstream group moves with constant velocity v_0, and the velocity of the downstream group oscillates around the constant velocity v_0. The oscillation amplitude increases with the increase of car index. That indicates that traffic jam appears in the noisy OV model.

Secondly, in order to control the traffic flow from jam to order, the feedback signal is designated as:

The feedback gain k is chosen according to equations (15) and (19). Substituting the parameter values into equations (15) and (19), and then $k \geq 0.5$ is obtained. Let us fix the feedback gain $k = 0.5$. Figure 3 exhibits the absolute values of the transfer function $\left|\bar{G}(j\omega)\right|$ as the function of the feedback gain k. From the Bode-plot, $\left\|\bar{G}(s)\right\|_\infty \leq 1$ when the feedback gain k is set to 0.5. The condition (15) is also satisfied, so the characteristic polynomial $\bar{d}(s)$ of $\bar{G}(s)$ is stable. Theoretically, no traffic jams occur in the OV model if the feedback signal with the gain $k = 0.5$ is introduced into system (2). The effect of the feedback signal on the added noise is studied on computer simulation.

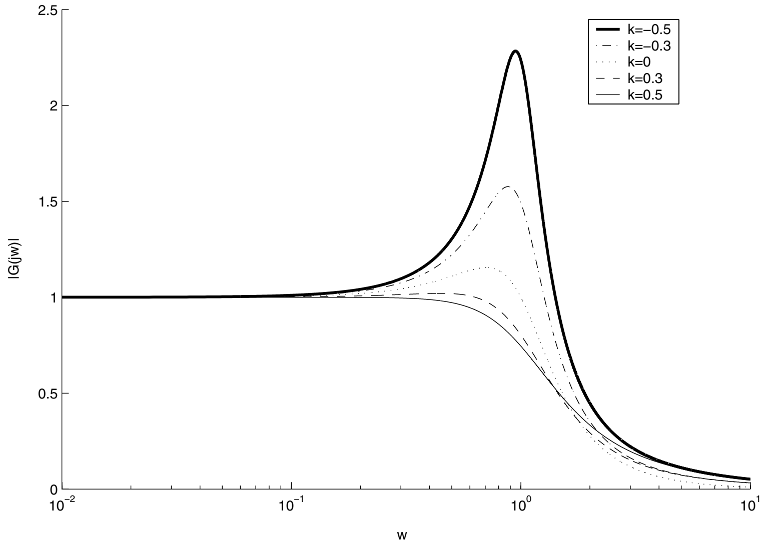

Fig. 3. Bode-plot for different values of feedback gain k.

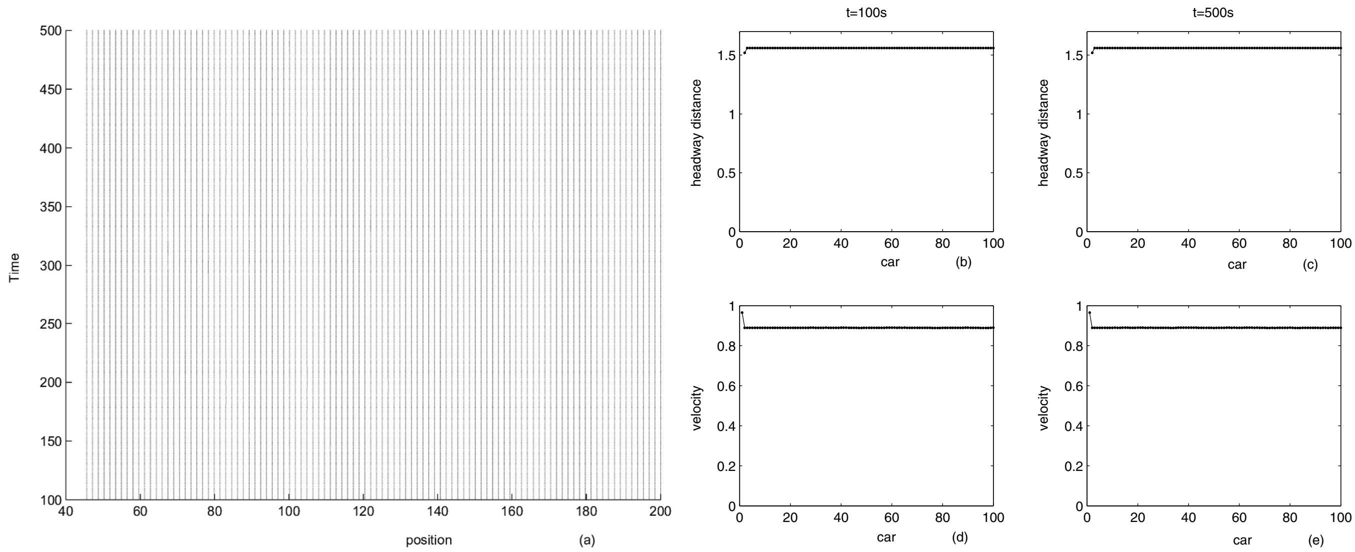

Fig. 4. (a) Spatio-temporal pattern of the traffic flow in the controlled OV model ($t = 100 \sim 500$) with random noise; (b-c) headway distance at $t = 100$ s and $t = 500$ s corresponding to Figure 4a; (d-e) velocity at $t = 100$ s and $t = 500$ s corresponding to Figure 4a.

Thirdly, we study the traffic flow in the controlled OV model with the random generating noise. Figure 4a shows the spatio-temporal pattern of the traffic flow under the effect of the feedback signal. From Figure 4a, it can be observed that the traffic flow is homogeneous in the controlled system, and all vehicles run orderly on the road. Furthermore, the corresponding traffic series of headway pattern at $t = 100$ and $t = 500$ placed in Figures 4b-c reveal order behaviors on the road. Figures 4d-e shows the variation of the velocity at $t = 100$ and $t = 500$ in the controlled OV model. It is obvious that the traffic flow is at order state when the feedback signal is introduced into the noisy OV model. The results indicate that traffic jams can be suppressed by a feedback control signal in a noisy OV model.

Finally, let us add a step external disturbance on the leader vehicle in the noisy OV model as follows

$$v_1(t) = \begin{cases} v_0/2, & \text{if} \quad 110 \le t \le 115 \\ v_0/2, & \text{if} \quad 130 \le t \le 135 \\ v_0, & \text{if} \quad \text{otherwise.} \end{cases} \tag{20}$$

Obviously, the step external disturbance shown in equation (20) is harmful. The generating noise is harmful to the

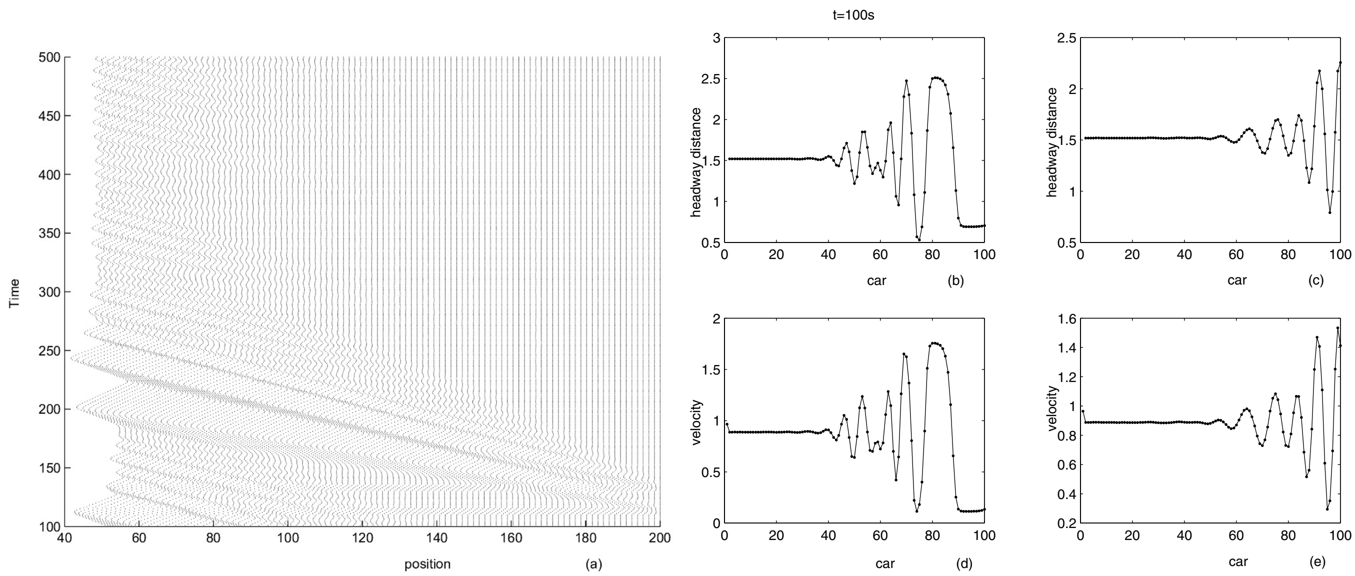

Fig. 5. (a) Spatio-temporal pattern of the traffic flow in the OV model ($t = 100 \sim 500$) with both external step disturbance on the leading vehicle and random noise; (b-c) headway distance at $t = 100$ s and $t = 500$ s corresponding to Figure 5a; (d-e) velocity at $t = 100$ s and $t = 500$ s corresponding to Figure 5a.

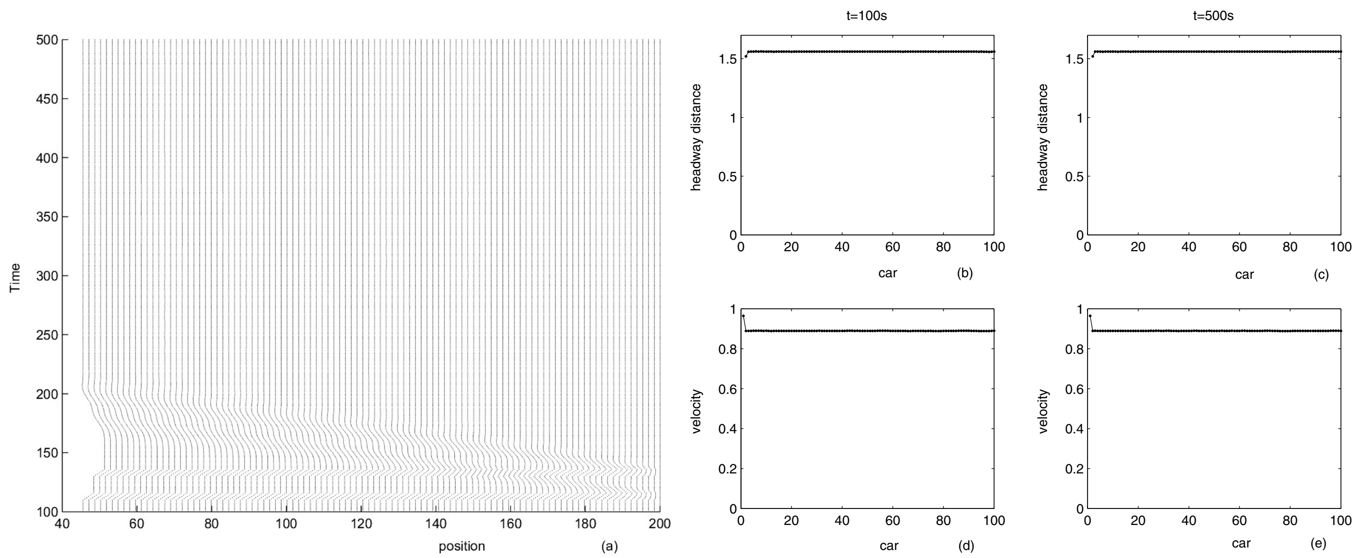

Fig. 6. (a) Spatio-temporal pattern of the traffic flow in the controlled OV model ($t = 100 \sim 500$) with both external step disturbance on the leading vehicle and random noise; (b-c) headway distance at $t = 100$ s and $t = 500$ s corresponding to Figure 6a; (d-e) velocity at $t = 100$ s and $t = 500$ s corresponding to Figure 6a.

traffic system. Figure 5a shows the spatio-temporal pattern of the traffic flow in the running traffic model with the step external disturbance. Figures 5b-c exhibits the headway distance at $t = 100$ and $t = 500$. Figures 5d-e shows the variation of the velocity at $t = 100$ and $t = 500$. From Figure 5, it can be seen that traffic jams still appear in the OV model when the control signal has not been added. Figure 6a and Figures 6b-c show the spatio-temporal pattern of the traffic flow and the headway distance in the controlled traffic model at $t = 100$ and $t = 500$, respectively. Figures 6d-e shows the variation of the velocity at $t = 100$ and $t = 500$. In Figure 6, it is obvious that the traffic system is at order state. From all above, it is confirmed that no traffic jams occurs in the noisy OV model if the feedback control signal has been used.

5 Conclusions and future works

The aim of this paper is to suppress the traffic jam by using the feedback control signal in the OV model. The signal acts in the system only if the state is unstable, and the signal vanishes in the stable system. Through theoretical analysis and numerical simulations, we conclude that:

(i) By using the feedback signal, the congested state in the traffic flow can be suppressed, and the vehicle traffic appears homogeneous phase.
(ii) Two conditions in Definition 1 [12], $\|G(s)\|_\infty > 1$ and stable $d(s)$, are unsuitable for sufficient and necessary conditions of traffic jam in system (5). In Section 2.3, this is illustrated by a case where traffic system is disturbed by a helpful disturbance and the two conditions are satisfied. The simulation results show that no traffic jam occurs in the system. Therefore, the definition of robustness maybe not directly be used for no traffic jam in the traffic system. While $\|G(s)\|_\infty \leq 1$ and stable polynomial $d(s)$ are really sufficient for no occurrence of traffic jam.
(iii) Base on (ii), sufficient conditions for no traffic jam in the controlled system (7) are derived in Theorem 1. The conditions provide theoretical foundations for numerical simulations.
(iv) Numerical simulations agree with our theoretical analysis. Our simulation results showed that the disorder state in the uncontrolled system induced by harmful disturbances disappeared when the control method carried out in OV model. The simulation results validate our theoretical analysis.

In this paper, we simply give the sufficient conditions for no traffic jam in the controlled system (7). Therefore, it would be interesting to study the necessary conditions. The further work also includes considering the definition of the traffic jams. This paper was partly supported by National Outstanding Young Investigation Grant (70225005) and Project (70471088) of National Natural Science Foundation of China, Foundation of Beijing jiaotong University (2003SM031, 2003SM032) and Rencai Foundation of Beijing jiaotong University (2003RC010), China.

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